Karunya University
(Karunya Institute of Technology and Sciences)
(Declared as Deemed to be University under Sec.3 of the UGC Act, 1965)

End Semester Examinations - 2015-16 Even Semester - May 2016

15MA3006 Linear Algebra

Set A

Time : 3 hrs
Total Marks: 100

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1. a) If W_1 and W_2 are finite dimensional subspace of a vector space V over F, then prove that W_1+W_2 is finite dimensional and $dim\ W_1 + dim\ W_2 = dim(W_1 \cap W_2) + dim(\ W_1 + W_2)$.

   b) Prove that the vectors $(1,1,0,0), (0,0,1,1), (1,0,0,4), (0,0,0,2)$ forms a basis for R^4. Also express each of the standard basis as linear combination of these vectors. (12+8)

OR

2. a) Prove that $T : R^2 \to R^3$ defined as $T(a,b) = (a+b, a-b, a)$ is a linear transformation. Also find the range, rank, null space and nullity of T.

   b) Let V and W be finite dimensional vector spaces over the field F such that $dim\ V = dim\ W$. If T is a linear transformation from V into W, then prove that the following are equivalent:

   i) T is invertible.

   ii) T is non-singular.

   iii) T is onto.

   iv) If $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ is basis for V, then $\{T(\alpha_1), T(\alpha_2), ..., T(\alpha_n)\}$ a basis for W.

   v) There is some basis $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ for V such that $\{T(\alpha_1), T(\alpha_2), ..., T(\alpha_n)\}$is a basis for W. (14+6)

3. a) Let T be a linear operator on a finite-dimensional space V and c be a scalar. Then prove that the following are equivalent:

   i) c is a characteristic value of T.

   ii) The operator $(T - cI)$ is singular.

   iii) $det(T - cI) = 0$.

   b) Find the characteristic and minimal polynomials of $A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}$ (10+10)

OR

4. a) Let T be a linear operator on R^3 defined by

   $$T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3).$$

   i) What is the matrix form of T in the standard ordered basis for R^3.

   ii) What is the matrix form of T in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$, where $\alpha_1 = (1,0,1), \alpha_2 = (-1,2,1), \alpha_3 = (2,1,1)$.

5. a) Let $T : V \to V$ be linear operator. Suppose for $v \in V, T^k(v) = 0$, but $T^{k-1}(v) \neq 0$. Then prove that

i) The set $S = \{v, T(v), T^2(v), ..., T^{k-1}(v)\}$ is linearly independent.

ii) The subspace W spanned by S is T invariant.

iii) The restriction $\overline{T}$ of T to W is nilpotent of index k.

iv) The matrix of T relative to the basis $\{T^{k-1}(v), ..., T(v), v\}$ of W is of the form

$$\begin{pmatrix} 0 & 1 & 0 & . & . & 0 & 0 \\ 0 & 0 & 1 & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & 1 \\ 0 & 0 & 0 & . & . & 0 & 0 \end{pmatrix}$$

b) Suppose that the characteristic and minimal polynomial of a linear operator T are $Char(T) = (t-4)^4(t-3)^3$ and $m(T) = (t-2)^2(t-3)^2$ respectively. Find all possible Jordan canonical form of T. (12+8)

OR

6. a) Let V be a finite dimensional vector space over F and T be a linear operator on V. Prove that T is triangulable if and only if the minimal polynomial of T is a product of linear polynomials over F.

b) Let A and B be two square matrices of order n over a field F and $\lambda \in F$. Prove that

i) $trace(\lambda A) = \lambda trae(A)$.

ii) $trace(A + B) = trace(A) + trace(B)$.

iii) $trace(AB) = trace(BA)$. (10+10)

7. a) Find the possible Rational canonical form of 6×6 matrices with minimal polynomial $m(t) = (t+1)^3$.

b) Define Companion matrix and $T-$annihilator of a vector α in a vector space V. (12+8)

OR

8. a) Prove that the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 + 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ is positively definite.

b) Find the matrix of the quadratic form $3x^2 - 5y^2 + 3z^2 - 3yz - 5zx + 7xy$. (10+10)

9. a) State and Prove Sylvester's Law.

b) Define Rank and Signature. (15+5)

Wishing you All the Best
